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Origin of Radiation Reaction

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The emission of radiation from an accelerated charge is analyzed. It is found that at zero velocity, the radiation emitted from the charge imparts no counter momentum to the emitting charge, and no radiation reaction force is created by the radiation. A reaction force is created by the stress force that exists in the curved electric field of the charge, and the work done in overcoming this force is the source of the energy carried by the radiation.

1. INTRODUCTION

Radiation emitted by an accelerated charge carries energy generated through the process of the creation of the radiation. Comparing accelerations of neutral and charged particles having equal masses, we find that for both particles, the work done by the accelerating force creates the kinetic energy of the particles. However, the acceleration of the charged particle generates additional energy, the energy carried by the radiation, and this additional energy is generated by an additional work done by the accelerating force. Thus the accelerating force of a charged particle may be decomposed into two parts: the first is the force that works against inertia, which generates the kinetic energy of the particle, and the second is the force that generates the energy carried by the radiation, which works against a reaction force, which is usually called, the "radiation reaction force" [1–3]. It is assumed that when radiation is emitted by an accelerated charge, a reaction force is created which opposes the acceleration, and the work done in overcoming this force is the source of the energy carried by the radiation.

However, not all the radiated power creates a reaction force, and when the velocity of the accelerated charge is very low this reaction force vanishes.

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What is then the source of the energy carried by the radiation? We find that, when a charged particle is accelerated in a free space, its electric field, which is an independent physical entity, is not accelerated with the charge. The electric field is curved [4], and a stress force exists between the accelerated charge and its curved electric field [5]. This force is (a part of) the reaction force, and the work done in overcoming this force is the source of the energy carried by the radiation. If the velocity of the accelerated charge is not zero, the radiation creates part of the reaction force, which together with the stress force of the curved electric field forms the total radiation reaction force.

In Section 2 we present the problem that arises when the *radiation* reaction force vanishes. In Section 3 we present the solution to the problem, where a reaction force is shown to exist due to the stress force in the curved electric field of the charge even at zero velocity. In Section 4 we calculate the reaction force for nonzero velocity. We conclude in Section 5.

2. THE PROBLEM

The formula for the angular distribution of the radiation power is [6]

$$\frac{dP}{d\Omega} = \frac{e^2 a^2}{4\pi c^3} \frac{\sin^2\theta}{(1 - \beta \cos \theta)^5} \tag{1}$$

where e is the accelerated charge, a is its acceleration, θ is the angle measured from the direction of the acceleration, and β is the velocity of the particle divided by the speed of light c.

Integrating Eq. (1) over the angles yields

$$P = \frac{2}{3} \frac{e^2(\gamma^3 a)^2}{c^3} \tag{2}$$

where $\gamma^2 = 1/(1 - \beta^2)$. Equation (2) yields the Larmor formula for the power carried by radiation for zero velocity ($\beta \to 0$):

$$P = \frac{2}{3} \frac{e^2 a^2}{c^3} \tag{3}$$

We plot in Fig. 1 the angular distribution of the radiation according to Eq. (1) for several values of β (a similar figure is given in ref. 6, Fig. 14.3).

It is clearly observed that for low velocities ($\beta \leq 0.01$) most of the radiation is emitted at a right angle to the direction of motion, and due to the symmetry in the plane perpendicular to the direction of motion, no counter momentum is imparted to the emitting particle and no reaction force is created by the radiation. What is the source of the energy carried by the radiation? This problem, the so-called “energy balance paradox,” has received several answers.

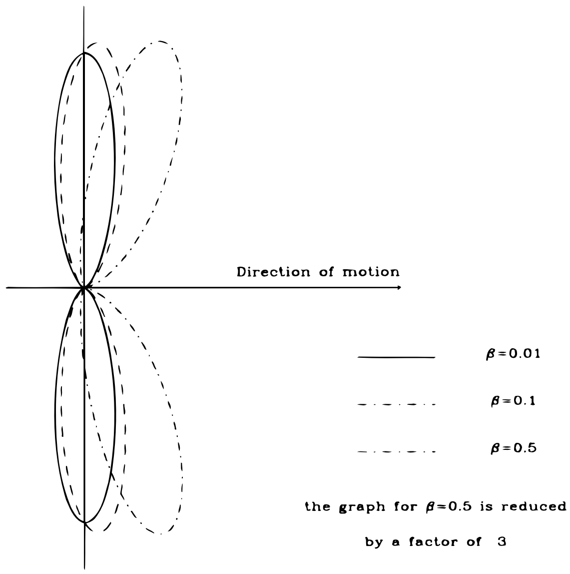

Fig. 1. The angular distribution of the radiation. The graph for $\beta = 0.5$ is reduced by a factor of 3.

One of them [7] is that there exists a charged plane whose charge is equal and opposite in sign to the accelerated charge and which recedes with the speed of light in a direction opposite to the direction of the acceleration. The interaction between this charged plane and the accelerated charge creates the energy carried by the radiation. Another suggestion [2] is that in such a case the energy radiated is supplied from the self-energy of the charge. These suggestions are far from being satisfactory. It should be noted that the idea suggested in ref. 7 resolves another difficulty concerning this topic, which is the existence of a single electric charge. As we assume that the matter in the universe is neutral, the existence of a solitary charge is a local phenomenon whose validity is limited to distance scales that are much shorter than distance scales that characterize gravitational considerations. Any treatment of this topic that carries calculations to infinity cannot be a valid treatment. The treatment suggested by Leibovitz and Peres [7] considers a system which is neutral.

The simplest case of acceleration is the one in which the particle is accelerated with a constant acceleration a in its own system of reference. In such a motion the particle is always at rest in a system of reference which is adjacent to the particle. Such a motion is characterized as a hyperbolic motion [8]. In such a motion the radiation reaction force calculated from the Dirac equation of motion vanishes for zero velocity, exactly as we observe in Fig. 1. It becomes evident that this reaction force cannot create the energy carried by the radiation, and the source of this energy should be looked for elsewhere.

3. THE SOLUTION

An electric field is an independent physical entity whose behavior depends on the properties of space in which it is induced. The electric field is induced by an electric charge, but once it is induced, its properties no longer depend on the charge that induced it. The electric field is inertial with respect to the local (free) system of reference, characterized by the geodesics that cover this system. Thus, when an accelerating charge induces an electric field, the field *is not accelerated* with the charge, and there exists a relative acceleration between the field and the charge that induced the field. (The existence of an acceleration between the charge and its electric field is the condition for the creation of radiation, and not the relative acceleration between the charge and the observer as suggested by several authors [5]). Let us consider the case of a constant acceleration, characterized as a hyperbolic motion.

The field induced by the accelerating charge is curved, and this curvature is displayed in Fig. 2 (this graph is similar to the one given by Singal [9]).

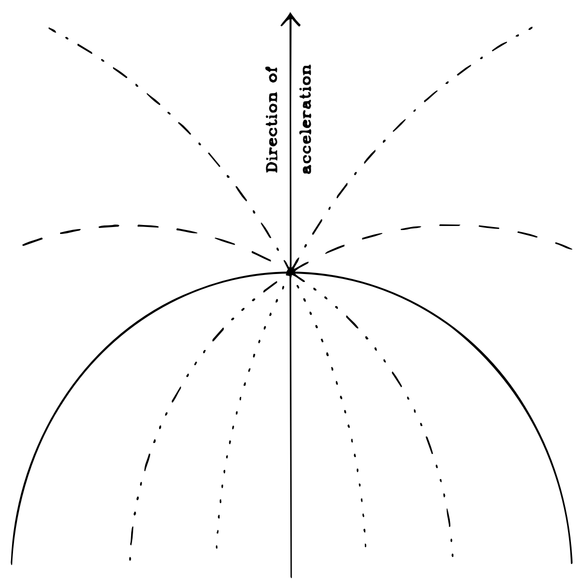

Fig. 2. The curved electric field of a uniformly accelerated charge.

Due to the curvature there exists a stress force in the field, whose force density is given by

$$f_E = \frac{E^2}{4\pi R_c} \tag{4}$$

where E is the field strength and R_c is the radius of curvature of the field. The interaction between the accelerated charge and its curved electric field creates a force that contradicts the acceleration. In order to overcome this force, the accelerating force should perform additional work (in addition to the work that creates the kinetic energy of the particle), and this additional work creates the energy carried by the radiation. We have to sum over the stress force f_E and calculate the work done against this force.

In order to sum over f_E, we have to integrate over a sphere whose center is located on the charge. Naturally, such an integration involves a divergence (at the center). To avoid such a divergence, we take as the lower limit of the integration a small distance from the center $r = c\Delta t$ (where Δt is infinitsimal). We calculate the work done by the stress force in the volume defined by $c\Delta t < r < r_{\text{up}}$, where $c^2/a \gg r_{\text{up}} \gg c\Delta t$. These calculations are performed in a system of reference S, defined by the geodesics, and momentarily coincides with the frame of reference of the accelerated charge at time $t = 0$ at the charge location (see ref. 5).

The force per unit volume due to the electric stress is $f_E = E^2/(4\pi R_c)$. The radius of curvature is calculated by using the formulas for the field lines [9]. It can be easily shown that in the limit of $a\Delta t \ll c$ the radius of curvature of a field line is $R_c \simeq c^2/(a \sin\theta)$, where θ is the angle between the initial direction of the field line and the acceleration (see Fig. 2). The force per unit volume due to the electric stress is

$$f_E(r) = \frac{E^2(r)}{4\pi R_c} = \frac{a \sin\theta}{c^2} \frac{e^2}{4\pi r^4} \tag{5}$$

where in the second equality we have substituted for the electric field $E = e/r^2$, which is a good approximation for low acceleration [4]. The stress force is perpendicular to the direction of the field lines, so that the component of the stress force along the acceleration is $-f_E(r) \sin\phi$, where ϕ is the angle between the local field line and the acceleration. For very short intervals (where the direction of the field lines does not change much from their original direction) $\phi \sim \theta$, and we can write

$$-f_E(r) \sin\phi \simeq -f_E(r) \sin\theta = \frac{-a \sin^2\theta}{c^2} \frac{e^2}{4\pi r^4}$$

The dependence of this force on θ is similar to the dependence of the radiation distribution of an accelerated charge at zero velocity on θ. Since the calculations are performed in the system S, which is a flat-space system (defined by the geodesics), the integration can be carried without using any terms concerning space curvature. Integration of the stress force over a spherical shell extending from $r = c\Delta t$ to $r_{\rm up}$, where $c^2/a \gg r_{\rm up} \gg c\Delta t$, yields the total force due to stress

$$F_E(t) = 2\pi \int_{c\Delta t}^{r_{\rm up}} r^2\, dr \int_0^{\pi} \sin\theta\, d\theta\, [-f_E(r)\sin\theta] = -\frac{2}{3}\frac{a}{c^2}\frac{e^2}{c\Delta t}\left(1 - \frac{c\Delta t}{r_{\rm up}}\right) \tag{6}$$

Clearly the second term in the parentheses can be neglected. The power supplied by the external force on acting against the electric stress is $P_E = -F_E v = -F_E a\Delta t$, where we substituted $v = a\Delta t$ and v is the charge velocity at time $t = \Delta t$ in the system S, the system defined by the geodesics, which mometarily coincides with the system of reference of the accelerated charge at time $t = 0$ at the charge location. Substituting for F_E, we obtain

$$P_E(t) = \frac{2}{3}\frac{a^2e^2}{c^3} \tag{7}$$

This is the power radiated by an accelerated charged particle at zero velocity [Eq. (3)].

Actually, the expression in Eq. (6) before substituting the limits of the integral $[F_{(E)} = -\frac{2}{3}ae^2/c^2r]$ equals the inertial force ($4m_e a/3$) of the electromagnetic mass of the charge as calculated by Lorentz (see ref. 6, p. 790). However, a work done against an inertial force cannot be a source of the energy carried by the radiation because such work creates a kinetic energy of the electromagnetic mass m_e and this leaves us with the energy balance paradox. The force F_E calculated in Eq. (6) is a stress force raised by the interaction between the curved electric field and the charge that induced the field, and the work done in overcoming this force is done in addition to any work done against inertial forces that create kinetic energy, and this work is the source of the energy carried by the radiation.

4. NONZERO VELOCITY

Equation (7) shows the power emitted by a uniformly accelerated charge calculated at zero velocity. We can consider the case of an accelerated charge moving with low velocity. From Fig. 1 we observe that part of the radiation is radiated forward, and this part of the radiation imparts a backward momentum to the radiating charge, thus creating a reaction force. To calculate the power created by *this* reaction force, we multiply Eq. (1) by $\cos\theta/c$ (to obtain the parallel component of the momentum flux of the radiation) and integrate over the angles. The integration yields for the parallel component of the momentum flux p_{par}

$$p_{par} = \frac{2}{3}\frac{e^2(\gamma^3 a)^2}{c^4}\beta \tag{8}$$

while the total absolute value of the momentum flux p is found by dividing Eq. (2) by c. (The perpendicular momentum flux vanishes because of the symmetry mentioned above, but we still can compare the parallel component of the momentum flux to the total absolute value of the momentum flux of the radiation.) By dividing p_{par} by p we find the weight of the parallel component of the momentum flux in the total absolute value of the momentum flux:

$$\frac{p_{par}}{p} = \beta \tag{9}$$

This fraction is the weight of the parallel component of the momentum flux of the radiation, and this fraction creates a reaction force: a radiation reaction force. To get the weight of the work done by the radiation reaction force we should multiply this fraction by βc (the velocity of the charge in the rest frame), and we find that the weight of the power done in overcoming the radiation reaction force (the reaction force created by the radiation) in the total power radiated is β^2. The other part of the energy $(1 - \beta^2 = 1/\gamma^2)$ is created by the stress force that exists in the curved electric field. We find that the weight of the work done by the stress force in the total work done by the total reaction force decreases when γ increases.

5. CONCLUSION

It is shown that for the case of an acceleration of a charge at zero velocity the emission of radiation by the accelerated charge does not create any reaction force, exactly as is found from Dirac's equation of motion for the electron. In such a case, the reaction force is the stress force created by the curved electric field, which does not participate in the acceleration of the charge. When the velocity of the accelerated charge is not zero, part of the radiation is emitted forward and contributes to the reaction force, while another part of the reaction force is contributed by the stress force in the curved electric field. The total reaction force is the sum of the contributions of these two parts.

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